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Code No. 17555/Core

FACULTY OF SCIENCE

M.Sc. IV - Semester Examination, July 2021

Subject: Mathematics / Applied Mathematics / Mathematics with Computer Science

Paper – I: Integral Equations and Calculus of Variations

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 Convert the initial value problem $y'' + (1 + x^2)y = \cos x,\ y(0) = 0,\ y'(0) = 2$ into an integral equation.

2 Find the resolvent kernel of Volterra integral equation with kernel $K(x,t) = 3^{x-t}$.

3 Show that $\beta(p,q) = \beta(p+1,q) + \beta(p,q+1)$.

4 Solve the integral equation $\phi(x) = \int_{-1}^{1} \frac{xt}{1+\phi^2(t)}\,dt$.

5 State and prove the fundamental lemma of calculus of variations.

6 Distinguish between strong variation and weak variation.

7 Find the extremals of the functional $v[y(x)] = \int_0^1 (1 + y''^2)\,dx$.

8 State and prove Hamilton's principle.

PART – B

Note: Answer any three questions. **(3 x 15 = 45 Marks)**

9 Solve the Volterra integral equation of the first kind $\int_0^x e^{x-t}\phi(t)\,dt = \sin x$ by reducing it into the Volterra integral equation of the second kind.

10 Explain the method of successive approximations, and use it to solve $\phi(x) = 1 + \int_0^x (x-t)\phi(t)\,dt$, $\phi_0(x) = 1$.

11 Find the characteristic numbers and eigen functions of the integral equation $\phi(x) = \lambda \int_{-1}^{1} (5xt^3 + 4x^2t + 3xt)\phi(t)\,dt$.

12 Solve the boundary value problem $y'' + y = x,\ y(0) = y(\pi/2) = 0$ using Green's function.

13 Derive the necessary condition for the functional $v[y(x)] = \int_a^b F(x, y, y')\,dx$ with the boundary conditions $y(a) = y_a,\ y(b) = y_b$ to have an extremum.

14 Explain the Brachistochrone problem and find a variational solution to it.

15 Derive the Euler-Poisson equation.

16 State and derive Hamilton's canonical equations.

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Code No. 17556/Core

FACULTY OF SCIENCE
M.Sc. IV - Semester Examination, July 2021

Subject: Mathematics
Paper – II: Elementary Operator Theory

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 Define point spectrum, continuous spectrum, residual spectrum and regular value of a linear operator on a normed space.

2 Prove that eigenvectors $x_1, x_2, \ldots, x_n$ corresponding to different eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ of a linear operator T on a vector space X constitute a linearly independent set.

3 Let T be compact linear operator and S a bounded linear operator on a normed space X. Then prove that TS and ST are compact.

4 Prove that every relatively compact subset B of a metric space X is totally bounded.

5 Let T:H→H and W:H→H be bounded linear operators on a complex Hilbert space H and
S = W*TW, then show that if T is self-adjoint and positive, so is S.

6 Prove that the sum of two positive operators is positive.

7 For any projection P on a Hilbert Space H, prove that
$\langle Px,x\rangle = \|Px\|^2$; $P \geq 0$, $\|P\| \leq 1$; $\|P\| = 1$ if $P(H) \neq \{0\}$.

8 Define projection on Hilbert space and spectral family associated with an operator T.

PART – B

Note: Answer any three questions. **(3 x 15 = 45 Marks)**

9 Prove that all matrices representing a given linear operator T:X→X on a finite dimensional normed space X relative to various bases for X have the same eigenvalues.

10 Show that the spectrum σ(T) of a bounded linear operator T on a complex Banach space X is closed.

11 Prove that (i) every compact linear operator T on a normed space X is bounded
(ii) The identity operator on infinite dimensional normed space X is not compact.

12 Let T:X→X be a compact linear operator on a normed space X and let $\lambda \neq 0$. Then prove that Tx-λ x=y (y$\in$ X given) has a solution x if and only if y is such that f(y)=0 for all $f \in X'$ satisfying $T^{\times} f - \lambda f = 0$.

13 Let T be a bounded self-adjoint linear operator on a complex Hilbert space H, H$\neq$ {0} and $m = \inf_{\|x\|=1} \langle Tx, x\rangle$, $M = \sup_{\|x\|=1} \langle Tx, x\rangle$, then prove that m and M are spectral values of T.

14 Let T:H→H be a bounded self-adjoint linear operator on a complex Hilbert space H, then prove that a number λ belongs to the resolvent set ρ (T) of T if and only if there exists a C>0 such that for every $x \in H, \|T_\lambda x\| \geq C\|x\|$.

15 Let P_1 and P_2 be projections on a Hilbert space H. Then prove that

(i) The sum $P = P_1 + P_2$ is a projection on H if and only if $Y_1 = P_1(H)$ and $Y_2 = P_2(H)$ are orthogonal.

(ii) If $P=P_1+P_2$ is a projection, P projects H onto $Y=Y_1 \oplus Y_2$.

16 State and prove the theorem on Spectral family associated with a bounded self-adjoint linear operator on a complex Hilbert space.

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Code No. 17557/Core

FACULTY OF SCIENCE
M.Sc. IV - Semester Examination, July 2021

Subject: Mathematics
Paper – III: Analytic Number Theory

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 For all $x \geq 1$ $\sum_{n \leq x} d(n) = x \log x + (2c-1)x + O(\sqrt{x})$.

2 For $x \geq 1$, prove that $\sum_{n \leq x} \phi(n) = \frac{3}{\pi^2} x^2 + O(x \log x)$.

3 For $x > 0$, prove that $0 \leq \frac{\psi(x)}{x} - \frac{I(x)}{x} \leq \frac{(\log x)^2}{2\sqrt{x} \log 2}$.

4 State and prove Abel's identity.

5 Prove that $\lim_{x \to \infty} \left(\frac{M(x)}{x} - \frac{H(x)}{x \log x} \right) = 0$.

6 For all $x \geq 1$ prove that $\sum_{p \leq x} \frac{\log p}{p} = \log x + O(1)$ where p is a prime. Also prove that there exist constants c_1 and c_2 such that $I(x) \leq c_1 x$ for all $x \geq 1$ and $I(x) \leq c_2 x$ for all sufficiently large x.

7 If G is finite group and $a \in G$ prove that there is a positive integer $n \leq |G|$ such that $a^n = e$.

8 Let A* denote the conjugate transpose of the matrix A. Then prove that AA* = nI where I is $n \times n$ identity matrix. Hence prove that n^{-1}A* is inverse of A.

PART – B

Note: Answer any three questions. **(3 x 15 = 45 Marks)**

9 (a) State and prove Legendre's identity.

(b) For $x \geq 2$, prove that $\sum_{p \leq x} \left[\frac{x}{p} \right] \log p = x \log x + O(x)$.

10 (a) If $x \geq 1$ and $\alpha > 0, \alpha \neq 1$, prove that $\sum_{n \leq x} \sigma_\alpha(n) = \frac{\zeta(\alpha+1)}{\alpha+1} x^{\alpha+1} + O(x^\beta)$ where $\beta = Max\{1, \alpha\}$.

(b) For $x \geq 2$, prove that $\log[x]! = x \log x - x + O(\log x)$ and hence prove that

$$\sum_{n \leq x} \wedge(n) \left[\frac{x}{n} \right] = x \log x - x + O(\log x).$$

11 (a) Prove that the following relations are logically equivalent:

(i) $\lim_{x\to\infty} \frac{\pi(x)\log x}{x} = 1$ (ii) $\lim_{x\to\infty} \frac{I(x)}{x} = 1$ (iii) $\lim_{x\to\infty} \frac{\psi(x)}{x} = 1$.

(b) For $n \geq 1$, prove that n^{th} prime p_n satisfies the in equalities

$$\frac{1}{6} n \log n < p_n < 12\left(n \log n + n \log \frac{12}{e} \right).$$

12 For $x \geq 2$, prove that $I(x) = \pi(x)\log x - \int_2^x \frac{\pi(t)}{t}dt$ and $\pi(x) = \frac{I(x)}{\log x} + \int_2^x \frac{I(t)}{t\log^2 t}dt$.

13 State and prove Selberg's asymptotic formula.

14 Prove that there is a constant A such that $\sum_{p\leq x} \frac{1}{p} = \log \log x + A + O\left(\frac{1}{\log x}\right)$.

15 Prove that a finite abelian group G of order n has exactly n distinct characters.

16 Let χ be any non-principal character modulo k and let f be a non-negative function which has a continuous negative derivative $f'(x)$ for $x \geq x_0$. Then prove that, if $y \geq x \geq x_0$, we have $\sum_{x<n\leq y} \chi(n)f(n) = O(f(x))$. If in addition,

$f(x) \to 0$ as $x \to \infty$ then prove that the infinite series $\sum_{n=1}^{\infty} \chi(n)f(n)$ converges and we have,

$$\sum_{n\leq x} \chi(n)f(n) = \sum_{n=1}^{\infty} \chi(n)f(n) + O(f(x)) \text{ for } x \geq x_0.$$

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Code No. 17560 / CORE

FACULTY OF SCIENCE

M.Sc. IV-Semester Examination, July 2021

Subject: Mathematics / Mathematics with Computer Science

Paper – IV : Cryptography

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any five questions. **(5x7=35 Marks)**

1. Prove that if a and b are units modulo , then ab is also a unit modulo m.

2. Suppose a and b are positive integers. If u and v are integers such that $au + bv = 1$, then prove that $\gcd(a, b) = 1$.

3. Define Discrete Logarithm Problem.

4. Prove that $N^{10}2^N = O(e^N)$.

5. Solve the simultaneous system of congruences:
   $x \equiv 37 \pmod{43}$, $x \equiv 22 \pmod{49}$, and $x \equiv 18 \pmod{71}$.

6. When do you say that an integer a is a witness for compositeness of an integer n? Give an example to explain the same.

7. Define elliptic curve discrete logarithm problem (ECDLP).

8. Let E be the elliptic curve $E: y^2 - 2x + 4$ and let $P(0,2)$ and $Q(3,-5)$.Then compute $P \oplus Q$.

PART – B

Answer any three questions. **(3x15=45 Marks)**

9. Let $m \geq 1$ be an integer. Then prove the following statements:

   (a) *If* $a_1 \equiv a_2 \pmod{m}$ *and* $b_1 \equiv b_2 \pmod{m}$, *then*

   $$a_1 \pm b_1 \equiv a_2 \pm b_2 \pmod{m} \quad \textit{and} \quad a_1 \cdot b_1 \equiv a_2 \cdot b_2 \pmod{m}.$$

   (b) *Let* a *be an integer. Then*

   $$a \cdot b \equiv 1 \pmod{m} \textit{ for some integer } b \textit{ if and only if } \gcd(a, m) = 1.$$

10. State and prove the fundamental theorem of arithmetic.

11. State and prove collision algorithm.

12. Explain the El Gamal Public Key Cryptosystem.

13. Explain the processes of Key creation, Encryption and Decryption in RSA Public Key Cryptosystem.

14. State the Pohlig-Hellman algorithm and explain the same.

15. Explain various stages of Elliptic Diffie-Hellman Key exchange problem .

16. Suppose that the cubic polynomial $X^3 + AX + B$ factors as

$$X^3 + AX + B = (X - e_1)(X - e_2)(X - e_3).$$

Then prove that $4A^3 + 27B^2 = 0$ if and only if $e_1 = e_2 = e_3$.

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Code No. 17562/Core

FACULTY OF SCIENCE
M.Sc. IV - Semester Examination, July 2021

Subject: Mathematics / Applied Mathematics
Paper – V (B) : Advance Operation Research

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any five questions. **(5 x 7 = 35 Marks)**

1 Define (i) Competitive game (ii) Pure strategies (iii) Mixed strategies.
2 Briefly explain the general rules for dominance.
3 Explain (i) Holding cost (ii) Set-up costs.
4 Explain the importance of ABC analysis in the problem of inventory control.
5 Obtain the necessary and sufficient conditions for the optimum solution of NLPP:
MinZ= $f(x_1, x_2) = 3c^{2x_1+1} + 2e^{x_2+5}$ STC $x_1 + x_2 = 7$ & $x_1, x_2 \geq 0$.
6 Write general and canonical forms of Non-linear programming problem.
7 Derive Kuhn-Tucker necessary conditions for an optimal solution to a quadratic programming problem.
8 Describe briefly the Beale's method for solving Quadratic programming problem.

PART – B

Note: Answer any three questions. **(3 x 15 = 45 Marks)**

9 Use dominance principle to reduce the following game to 2x2 game and hence solve them

$$\begin{bmatrix} 8 & 5 & 8 \\ 8 & 6 & 5 \\ 7 & 4 & 5 \\ 6 & 5 & 6 \end{bmatrix}.$$

10 Use graphical method to reduce the following game and hence solve

Player A

Player B $\begin{bmatrix} 1 & 3 & -1 & 4 & 2 & -5 \\ -3 & 5 & 6 & 1 & 2 & 0 \end{bmatrix}$

11 Derive the EOQ model without shortage when the economic lot size system with uniform demand.

12 Derive the EOQ model when shortages are allowed and with Constant Rate of demand, scheduling time constant.

13 Solve the non-linear programming problem

$$MinZ = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$$
$$STC \quad x_1 + x_2 + x_3 = 11 \quad and \quad x_1, x_2, x_3 \geq 0$$

..2..

14 State and prove Kuhn-Tucker necessary and sufficient conditions in NLPP.

15 Use Wolfe's method to solve the QPP:

$$Max\ Z = 8x_1 + 10x_2 - x_1^2 - x_2^2$$

$$STC \quad 3x_1 + 2x_2 \le 6$$

$$and \quad x_1, x_2 \ge 0$$

16 Solve the following QPP by Beale's method

$$Max\ Z = 2x_1 + 3x_2 - x_1^2$$

$$STC \quad x_1 + 2x_2 \le 4$$

$$x_1, x_2 \ge 0$$

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